Benha University **Faculty of Engineering – Shoubra** **Engineering Mathematics & Physics** **Department – Preparatory year**		**Final Term Exam** **Physics** **Date: 11/ 5 / 2019** **Duration: 3 hours**

Model Answer

<u>Question (1) (15 marks)</u>

a) In terms of the predictions of kinetic theory of an ideal gas, derive the expression gives the internal energy of n-moles and its dependence on gas temperature.

Solution

The kinetic energy if a molecule of mass m, and moving with root mean square velocity is

$$(K.E)_{mlecule} = \frac{1}{2}mv^2 = \frac{3}{2N}(\frac{1}{3}mv_{r.m.s}{}^2)$$

$$= \frac{3}{2N}PV$$

$$= \frac{3}{2N}RT$$

If the quantity of the gas is one mole then $N=N_A$

$$(K.E)_{molecule} = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

$$(K.E)_{molecule} = \frac{3}{2}KT$$

Also the kinetic energy for one mole of an ideal gas is the sum of all molecular energy then

$$(K.E)_{mole} = N_A \times (K.E)_{molecuie}$$

$$(K.E)_{mole} = N_A(\frac{3}{2}KT)$$

$$(K.E)_{mole} = \frac{3}{2}RT$$

And for n-moles

$$(K.E)_{n-mole} = \frac{3}{2} nRT$$

This means that the internal energy of a gas (transitional kinetic energy) is function only on the gas temperature and independent on the pressure and volume. An important result is obtained to describe the internal energy
$$U = \frac{3}{2} nRT$$

b) Derive the relation describe the root mean square velocity of ideal gas molecule and each of gas pressure and temperature.

Solution

the following expression for the pressure

$$P = \frac{Nmv_{rms}^2}{3V}$$

where *V* is the volume. Also, as *Nm* is the total mass of the gas, and mass divided by volume is density

$$P = \frac{1}{3}\rho\, v_{rms}^2$$

where ρ is the density of the gas.
$$v_{r.m.s} = \sqrt{\frac{3P}{\rho}}$$

If the quantity of the gas is one mole the $N=N_A$ and M is the molecular weight then

$$v_{r.m.s} = \sqrt{\frac{3RT}{M}}$$

c) Mass of 200 gm of helium at 70 ^{o}C is changed where pressure is doubled and volume decrease by 30% and temperature becomes 300 ^{o}C. Find: 1-final mass 2-intial and final number of molecules 3- intial and final root mean velocities and mean kinetic energies of molecules 4- intial and final internal energis of the two quantities of the gas

Solution

the gas is He of molar mass 4 gm

$$mass_1 = 200gm \qquad pressure\ P_1 \quad temp\ \ T_1 = 70 + 273 = 343K \qquad volume\ \ V_1$$

$$mass_2 = ?\,gm \qquad pressure\ P_2 = 2P_1 \quad temp\ \ T_2 = 300 + 273 = 573K \quad volume\ V_2 = 0.7V_1$$

$\therefore \quad PV = nRT = \frac{mass}{M} RT$ apply for two states and solve for second mass

1- $$mass_2 = mass_1 \left(\frac{P_2}{P_1}\right) \left(\frac{V_2}{V_1}\right) \left(\frac{T_1}{T_2}\right)$$

$$mass_2 = 200gm\ (2)\ (0.7)\ \left(\frac{343k}{573k}\right) = 167.6\ gm$$

2- The number of molecules N is calculated from the number of moles n where

$$N = n \times N_A = \left(\frac{mass}{M}\right) \times N_A$$

$$N_1 = n \times N_A = \left(\frac{mass_1}{M}\right) \times N_A = \left(\frac{200gm}{4gm}\right) \times 6.023 \times 10^{23} = 3.0115 \times 10^{25}\ molecule$$

$$N_2 = n \times N_A = \left(\frac{mass_2}{M}\right) \times N_A = \left(\frac{167.6gm}{4gm}\right) \times 6.023 \times 10^{23} = 2.523 \times 10^{25}\ molecule$$

3- the root mean squre velocity calculated from the equation

$$v_{r.m.s} = \sqrt{\frac{3RT}{M}}$$

$$v_1 = \sqrt{\frac{3RT_1}{M}} = \sqrt{\frac{3 \times 8.31 \times 343}{4 \times 10^{-3} Kg}} = 1462.1\ m/s$$

$$v_2 = \sqrt{\frac{3RT_2}{M}} = \sqrt{\frac{3 \times 8.31 \times 573}{4 \times 10^{-3} Kg}} = 1889.76\ m/s$$

4- the internal energy of molecules is calculated from the equation

$$U = \frac{3}{2} nRT$$

$$U_1 = \frac{3}{2} n_1 R T_1 = \frac{3}{2} \times \left(\frac{200gm}{4gm}\right) \times 8.31 \times 343 = 2.13 \times 10^5 J$$

$$U_2 = \frac{3}{2} n_2 R T_2 = \frac{3}{2} \times \left(\frac{167.6gm}{4gm}\right) \times 8.31 \times 573 = 2.99 \times 10^5 J$$

Question (2) (15 marks)

a) Derive an expression of the change in entropy for an ideal gas when under goes iso-thermal and iso-baric processes. Plot the P-V and T-S diagram for both processes.

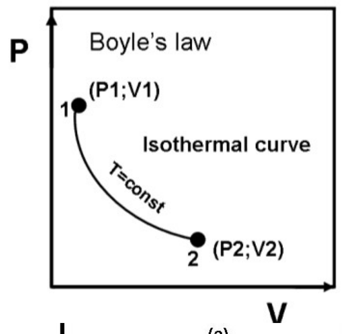
Solution

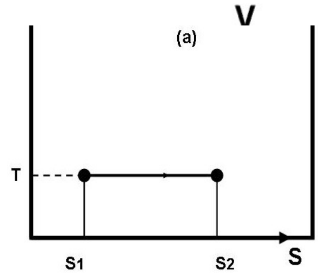
1-The change of entropy at isothermal processes:

In this process **T=const** and **dT=0**

dS= nR dV/V

$$\int_{S_1}^{S_2} dS = nR\int_{V_1}^{V_2} \frac{dV}{V}$$

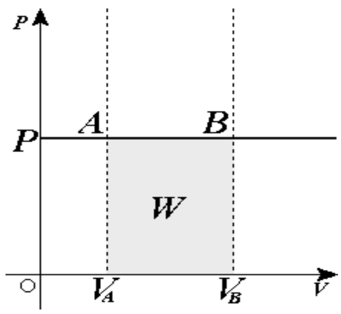




$$S_2 - S_1 = nR\ln\left(\frac{V_2}{V_1}\right)$$

or $$S_2 - S_1 = nR\ln\left(\frac{p_1}{p_2}\right)$$

2-The change of entropy at isobaric processes:

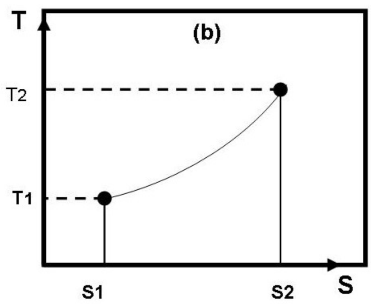


In this process P=constant and use the equation

dS= nC_v dT/T+ nR dV/V

its known before when p=const $$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

$$dS = \frac{dQ}{T} = \frac{nC_V dT}{T} + \frac{nRC_V dT}{T}$$



But $C_V+R=C_P$

$$\int_{S_1}^{S_2} dS = nC_p\int_{T_1}^{T_2} \frac{dT}{T}$$

$$\therefore \Delta S = S_2 - S_1 = nC_p\ln\left(\frac{T_2}{T_1}\right)$$

This equation can describe graphically as shown in figure.

b) An ideal gas initially at 300 K undergoes an isobaric expansion at 2.5 KPa. If the gas volume increases from 1 m^3 to 3 m^3 and if 12.5 KJ is transferred to the gas by heat. What are 1- change in internal energy 2- final temperature 3- change in gas entropy

Solution

The gas expanded at constant pressure or iso-baric process

$$T_1 = 300K \qquad P = Const = 2.5KPa \quad V_1 = 1m^3$$

$$T_2 = ? \qquad V_2 = 3m^3 \qquad dQ = 12.5KJ \qquad dU = ?$$

1- C hange of internal energy is calculated from first law of thermodynamics where

$$dU = dQ - dW$$

First the work must calculated as

$$dW = P(V_2 - V_1) = 2.5 \times 10^3\ Pa \times (3 - 1)m^3 = 5 \times 10^3\ J = 5KJ$$

2- The final temperature is obtained by using the iso-baric process where

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$or \qquad T_2 = T_1 \times \left(\frac{V_2}{V_1}\right) = 300\mathrm{K} \times \left(\frac{3\mathrm{m}^3}{1\mathrm{m}^3}\right) = 900\mathrm{K}$$

3- The change in entropy is calculated by knowing the heat added dQ and the temperature where

$$dS = \frac{dQ}{T} = \frac{12.5 \times 10^3 J}{300K} = 41.67\ J/K$$

c) In Carnot engine the entropy chnges by 25 C/K and the heat engine working between 27 °C and 200 °C. Calculate 1- efficiency of the engine 2- quntity of heat fllow in and fllow out the engine 3- the work done by the engine 4- how much heat must be added to do a work by 2 KJ.

Solution

$$\Delta S = 25\ \mathrm{C/K} \qquad T_1 = (27 + 273) = 300\mathrm{K} \quad T_2 = (200 + 273) = 573\mathrm{K}$$

1- efficiency of the engine is calculated from temperatures as

$$\eta = \frac{T_2 - T_1}{T_2} = \frac{573\mathrm{K} - 300\mathrm{K}}{573\mathrm{K}} = 0.301$$

2- quntity of heat fllow in and fllow out the engine are calculated using the equations

$$Q_{in} = T_h(S_2 - S_1) = T_h \times \Delta S \qquad \text{and} \qquad Q_{Out} = T_c(S_2 - S_1) = T_c \times \Delta S$$

$$Q_{in} = 573K \times 25\ \frac{c}{k} = 14325\ C = 14.325\ KC$$

and

$$Q_{out} = 300K \times 25\frac{c}{k} = 7500\ C = 7.5\ KC$$

3- the work done by the engine is calculated from

$$\eta = \frac{W}{Q_{in}}$$

And $$W = Q_{in} \times \eta = 14.325\ KC \times 0.301 = 4.311\ KC = 1.031KJ$$

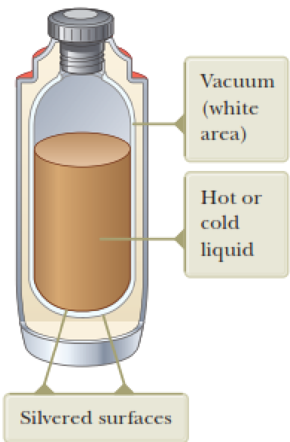
4- To calculate the heat must be added to do a work by 2 KJ use

$$Q_{in} = \frac{W}{\eta} = \frac{2KJ}{0.301} = 6.644KJ$$

<u>The third question (15 marks)</u>

A- a) The temperature inside the Dewar flask remains constant.**(2 Marks)**

Because the standard construction of Dewar flaskconsists of a double-walled Pyrex glass vessel with silvered walls (as shown in figure).The space between the walls is evacuated to minimize energy transfer by conduction and convection.The silvered surfaces minimize energy transfer by radiation because silver is a very good reflector and has very low emissivity. A further reduction in energy loss is obtained by reducing the size of the neck. So the liquid in that flask remains constant.



A- b)It is desirable to paint hot water pipes in thermal power station by aluminium paint.

Because aluminum paint is **bad absorber** for heat radiation .So it is a **bad emitter** for heat according to Kirchhoff's law.So water in pipes remains constant.

A-c) Firemen use fire screen made of glass not quartz.

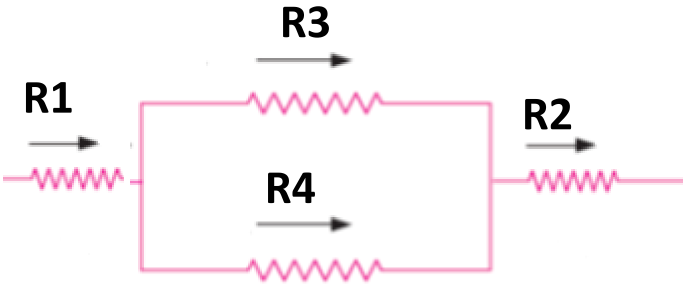
Because glass prevents a large portion of heat radiations emitted by fire.But quartz would allow most of these to pass through.

A-d)Outdoors in the winter, a piece of metal feels colder than a piece if wood.

Because metals transfer heat energy at higher rate than wood as the thermal conductivity coefficient is higher than that of wood ($K_{metal} > K_{wood}$).

3-B) The answer is shown in details in text book page no. (28-29)and(60-61). **(3 Marks)**

3-C) The equivalent electrical circuit is shown in the following figure:(6 Marks)



1- **To get the total thermal resistance:**

$$R_1 = \frac{X_1}{K_1 A_1} = \frac{0.1}{0.002\times2\times0.5} = 50$$

$$R_2 = \frac{X_2}{K_2 A_2} = \frac{0.1}{0.002\times2\times0.5} = 50$$

$$R_3 = \frac{X_3}{K_3 A_3} = \frac{2\times0.1}{1.5\times0.002\times2\times0.5} = 133.33$$

$$R_4 = \frac{X_4}{K_4 A_4} = \frac{2\times0.1}{2\times0.002\times0.5} = 100$$

$$R_{total} = R_1 + \frac{R_3 R_4}{K_3 + R_4} + R_2 = 157.1428$$

2-**To get the thermal current through the wall:**

$$q_{total} = \frac{T_1 - T_4}{\sum R} = 1.909\ watt$$

3- To get T_2 and T_3:

$$q_{total} = \frac{T_1 - T_2}{R_1} = 1.909\ watt$$

$$T_2 = 504.5℃$$

$$q_{total} = \frac{T_3 - T_4}{R_2} = 1.909\ watt$$

$$T_3 = 395.45℃$$

3-D) (4 Marks)

T=2177+273=2450 °K

ε=0.30, E=25 watt

$$E = \sigma \varepsilon A T^4$$

$$25 = 5.67 \times 10^{-8} \times 0.3 \times A \times 2450^4$$

$$A = 4.079 \times 10^{-5}\ m^2$$

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<u>Question 4 (15 marks)</u>

4-a) Define: (3 marks)

(i) **Thin Lens**: *It is a lens with a thickness (distance between the two refracting surfaces) is negligible compared to the radii of curvature of the lens surfaces*

(ii) **Population Inversion**: *The situation in which at least one of the higher energy levels has more atoms than a lower energy level.*

(iii) **Beer's Law**: *When a light passes through absorbing medium at right angle to the plane of surface or the medium or the solution, the rate of attenuation in the intensity of the transmitted light decreases exponentially as the medium thickness increases.*

4-b) **Figure (1)** depicts a simplistic optical fiber: a plastic core (**n_1=1.58**) is surrounded by a plastic sheath. Light rays can be incident on one end of the fiber at angle **θ=30°.** The ray is required to undergo total internal reflection at point ***A***, where it encounters the core–sheath boundary. What is the minimum value of sheath refractive index (**n_2**) that allows total internal reflection at ***A***? (4 marks)

Solution

n_1=1.58, θ=30°, n_2=??

From Snell's law

$$(1)\sin 30 = (1.58)\sin\beta$$

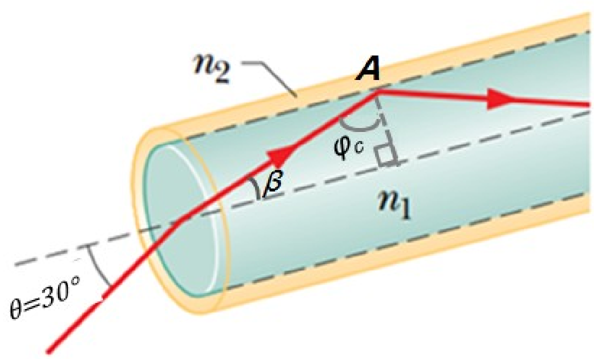
$$\beta = 18.4°$$

$$\varphi_C + \beta = 90°$$

$$\varphi_C = 90° - 18.4 = 71.55°$$

$$\sin\varphi_C = \frac{n_2}{n_1}$$

$$\sin 71.55 = \frac{n_2}{1.58}$$



$$n_2 = \mathbf{1.498}$$

4-c) Drive the expression that show the relation between the absorption coefficient (**α**) and the population of energy levels for an absorbing medium (4 marks)

The rate of loss of photons when the beam travels distance (ΔZ) through the gas is given by

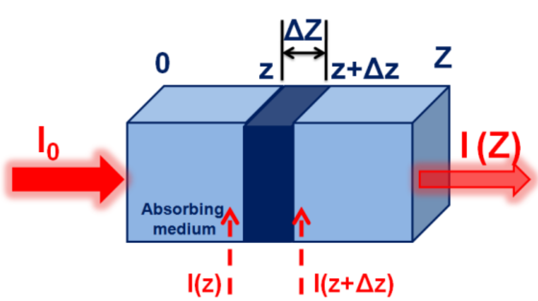
$$-\frac{dn}{dt} = N_1 B_{12}\rho - N_2 B_{21}\rho$$

$$\because B_{12} = B_{21} \textit{ and } \rho = \frac{I(Z)}{4\pi c}$$

$$\therefore -\frac{\boldsymbol{dn}}{\boldsymbol{dt}} = (\boldsymbol{N_1} - \boldsymbol{N_2})\boldsymbol{B_{12}}\frac{\boldsymbol{I(Z)}}{\boldsymbol{4\pi c}} \dots\dots\dots (\mathbf{1})$$

This rate can be also given by

$$-\frac{dn}{dt} = [I(Z) - I(Z+\Delta Z)]\frac{A}{h\nu}$$



$A \rightarrow$ *is the beam cross section area*

$$-\frac{dn}{dt} = \Delta I(Z)\frac{A}{h\nu}$$

$$-\frac{\boldsymbol{dn}}{\boldsymbol{dt}} = \boldsymbol{\alpha I(Z)\Delta Z}\frac{\boldsymbol{A}}{\boldsymbol{h\nu}} \ldots\ldots\ldots(\mathbf{2})$$

By equating (1) & (2) we get

$$\alpha \boldsymbol{I(Z)}\Delta Z\frac{A}{h\nu} = (N_1 - N_2)B_{12}\frac{\boldsymbol{I(Z)}}{4\pi c}$$

$$\therefore \alpha = B_{12}\frac{(N_1 - N_2)}{A\Delta Z}\frac{h\nu}{4\pi c}$$

$n_1 = \dfrac{N_1}{A\Delta Z} \rightarrow$ *is the number of atoms* ***per unit volume*** *in level* (E_1).

$n_2 = \dfrac{N_2}{A\Delta Z} \rightarrow$ *is the number of atoms* ***per unit volume*** *in level* (E_2).

$$\therefore \boldsymbol{\alpha = B_{12}(n_1 - n_2)\frac{h\nu}{4\pi c}}$$

4-d) If the spontaneous emission coefficient is **10^6 s^{-1}** for an x-ray wavelength transition of **100 nm**:

(i) What would be the corresponding stimulated emission coefficient?

(ii) What must be irradiance to cause stimulated emission three times greater than the spontaneous emission? [given ***h=6.625×10^{-34} J.s, c=3×10^8 m/s***] (4 marks)

Solution

$A_{21}=10^6$ s^{-1}, λ=100 nm, (i) B_{21}=??, (b) I=?? for st. emission=3×sp. Emission

(i)

$$\frac{\boldsymbol{A_{21}}}{\boldsymbol{B_{21}}} = \frac{\boldsymbol{8\pi h\nu^3}}{\boldsymbol{c^3}} = \frac{\boldsymbol{8\pi h}}{\boldsymbol{\lambda^3}}$$

$$\frac{10^6}{B_{21}} = \frac{8\pi(6.625 \times 10^{-34})}{(100 \times 10^{-9})^3}$$

$$B_{21} = \mathbf{6 \times 10^{16}}\ \boldsymbol{m^3/W.s^3}$$

(ii)

st. emission=3×sp. Emission

$$N_2 B_{21}\rho = 3 \times N_2 A_{21}$$

$$N_2 B_{21}\rho = 3 \times N_2 A_{21}$$

$$\therefore \rho = \frac{I}{4\pi c} = \frac{3A_{21}}{B_{21}}$$

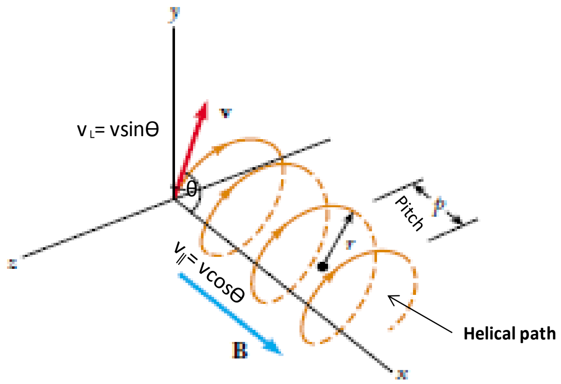
$$\frac{I}{4\pi(3 \times 10^8)} = \frac{3 \times 10^6}{6 \times 10^{16}}$$

$$I = \mathbf{0.188}\ \boldsymbol{watt/m^2}$$

Question (5) (15 Marks)

5-a) [5 Marks]

- If the charged particle moves in a uniform magnetic field with its velocity **at some arbitrary angle ϴ** with respect to B, **its path is a helix**



- The cross section area of the helix is a circle of radius $r = \frac{mv\perp}{qB} = \frac{mv\sin\theta}{qB}$

- Pitch of the helix (p)=distance travelled by the Particle a long the direction of (B)in one period $p = (v\cos\theta)\frac{2\pi m}{qB}$

5-b) [5 Marks]

i- The particle moves in a horizontal line (without deviation i.e. undeflected) through the fields if

$$qE = qvB \qquad \therefore B = \frac{E}{v} = \frac{10^5}{5x10^5} = 0.2T$$

ii- When the electric field is turned off the deuterons move only in a magnetic field in a cirlcle of radius r

$$r = \frac{mv}{qB} = \frac{3.34x10^{-27}x5x10^5}{1.6x10^{-19}x0.2} = 5.2x10^{-2}m$$

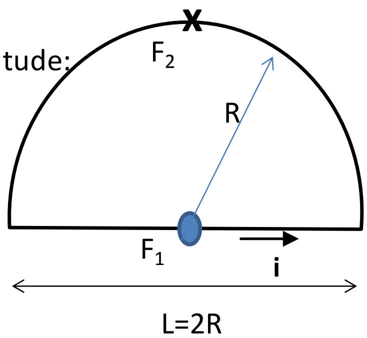
5-c) [5 Marks]

- The magnetic force F_1 acting on the straight portion has a magnitude:

F_1 =BiL=2BiR *Out of the page*

- The magnetic force F_2 on the curved portion is the same as that on a straight wire of length 2R carrying current i to the left.

F_2=2BiR *into the page*



The net magnetic force on the loop is $\sum F = F_1 + F_2 = 0$

Question (6) (15 Marks)

6-a) [5 Marks]

Derivation of coefficient of self inductance (L) of a solenoid

$$\because L = N\frac{\phi_B}{i}$$

$$L = N\frac{\mu_0 N i}{il}A$$

$$L = \frac{\mu_0 N^2}{l}A$$

$$\therefore L = \mu_0 n^2 l A$$

Self inductance length

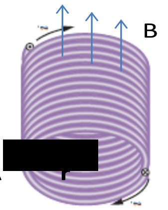
•The magnetic field inside a solenoid $B = \mu_0 n i = \frac{\mu_0 N i}{l}$

•The magnetic flux through each turn

$$\therefore \phi_B = BA = \frac{\mu_0 N i}{l}A$$

Where N is the number of turns

$n = \frac{N}{l}$ is the number of turns per unit length



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6-b) [5 Marks]

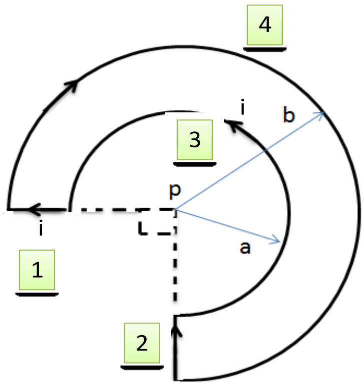
For radial straight wires

for section 1 The angle θ between dl and r^ is 180°

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin 180}{r^2} = 0 \Rightarrow B_1 = 0$$

for section 2 The angle θ between dl and r^ is zero

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin 0}{r^2} = 0 \Rightarrow B_2 = 0$$



For circular arcs

$$B = \frac{3}{4}\left(\frac{\mu_0 i}{2R}\right) = \frac{3\mu_0 i}{8R}$$

$$\therefore B_{3(out)} = \frac{3\mu_0 i}{8a} \qquad \therefore B_{4(in)} = \frac{3\mu_0 i}{8b}$$

$$\text{Net field } B = B_1 + B_2 + B_3 + B_4$$

$$B = 0 + 0 + \frac{3\mu_0 i}{8a} - \frac{3\mu_0 i}{8b}$$

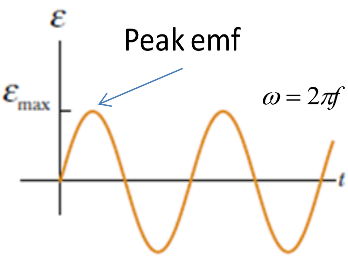
$$B = \frac{3\mu_0 i}{8}\left(\frac{1}{a} - \frac{1}{b}\right)$$

6-c) [5 Marks]

$$\varepsilon_{max} = NAB\omega = NAB(2\pi f)$$

$$N = \frac{\varepsilon_{max}}{AB(2\pi f)} = \frac{48}{(0.1x0.1)x0.64x(2\pi x50)} = 24 \text{ turn}$$

$NBA\omega = \varepsilon_{max}$



$\omega = 2\pi f$ rad/sec