Benha University **Faculty of Engineering – Shoubra** **Preparatory Year for Master of Science in Engineering Physics**		**Final Term Exam** **Plasma Physics** **Date: 21 / 1 / 2015** **Duration : 3 hours**

- Answer all the following question
- Illustrate your answers with sketches when necessary
- No. of questions: 4 in two pages
- Total Mark: 200 Marks

Physical constants: Charge of an electron, $e = 1.6 \times 10^{-19}$ C
Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg
Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg
Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J K^{-1}
Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Question (1) (45 marks)

(A) What does "plasma" mean as a state of matter? Give some examples of the common forms of plasma? **(15 marks)**

Answer: In physics and chemistry, **plasma** is a state of matter similar to gas in which a certain portion of the particles are ionized. The basic premise is that heating a gas dissociates its molecular bonds, rendering it into its constituent atoms. Further heating leads to ionization (a loss of electrons), turning it into a plasma: containing charged particles, positive ions and negative electrons.

The presence of a non-negligible number of charge carriers makes the plasma electrically conductive so that it responds strongly to electromagnetic fields. Plasma, therefore, has properties quite unlike those of solids, liquids, or gases and is considered to be a distinct state of matter. Like gas, plasma does not have a definite shape or a definite volume unless enclosed in a container; unlike gas, under the influence of a magnetic field, it may form structures such as filaments, beams and double layers. Some common plasmas are stars and neon signs.

Examples

It has been said that 99% of matter in the universe is in the plasma state.

Lightning, earth's ionosphere, interplanetary medium, solar wind, solar corona

Laboratory plasmas such as glow discharges, arcs, fluorescent lamps, thermonuclear fusion

(B) Compare the main characteristics which differentiate the plasma state and ordinary gaseous phase of matter? **(15 marks)**

Answer:

Property	Gas	Plasma
Electrical Conductivity	**Very low** Air is an excellent insulator until it breaks down into plasma at electric field strengths above 30 kilovolts per centimeter.	**Usually very high** For many purposes, the conductivity of a plasma may be treated as infinite.
Independently acting species	**One** All gas particles behave in a similar way, influenced by gravity and by collisions with one another.	**Two or three** Electrons, ions, protons and neutrons can be distinguished by the sign and value of their charge so that they behave independently in many circumstances, with different bulk velocities and temperatures, allowing phenomena such as new types of waves and instabilities.
Velocity distribution	**Maxwellian** Collisions usually lead to a Maxwellian velocity distribution of all gas particles, with very few relatively fast particles.	**Often non-Maxwellian** Collisional interactions are often weak in hot plasmas and external forcing can drive the plasma far from local equilibrium and lead to a significant population of unusually fast particles.
Interactions	**Binary** Two-particle collisions are the rule, three-body collisions extremely rare.	**Collective** Waves, or organized motion of plasma, are very important because the particles can interact at long ranges through the electric and magnetic forces.

(C) Consider a plasma in the Plasma Department's TORTUS tokamak where $n_e = n_i = 10^{19}$ m^{-3}, $T = 1.16 \times 10^6$ K and volume of plasma = 1 m^3. How much would the energy in this plasma raise the temperature of 150 ml of water?

[Density of water = 1000 kg/m^3 , Specific heat of water = 4200 J/kg K] **(15 marks)**

Answer:

$n_o = n_e = n_i = 10^{19}$

Volume of plasma = 1 m^3

$T = 1.16 \times 10^6$ K

Volume of water = 150×10^{-6} m^3

The average energy per particle,

$$E_{av} = \frac{3}{2} kT$$

$= 1.5 \times 1.38 \times 10^{-23} \times 1.16 \times 10^6$

$= 2.4 \times 10^{-17}$ J

The total energy in 1 m^3 = $10^{19} \times 2.4 \times 10^{-17}$

= 240 J

Q(lost by plasma) = Q(gained by water)

$240 = m\, c\, \Delta T$

$240 = \rho\, V\, c\, \Delta T$

$240 = 1000 \times 150 \times 10^{-6} \times 4200\ \Delta T$

$\Delta T = 0.38\ ^{o}C$

Question (2) **(50 marks)**

(A) Give short notes on: **(20 marks)**

1) The plasma approximation.

Charged particles must be close enough together that each particle influences many nearby charged particles, rather than just interacting with the closest particle (these collective effects are a distinguishing feature of a plasma). The plasma approximation is valid when the number of charge carriers within the sphere of influence (called the *Debye sphere* whose radius is the Debye screening length) of a particular particle are higher than unity to provide collective behavior of the charged particles.

2) Bulk interaction of plasma.

The Debye screening length (defined above) is short compared to the physical size of the plasma. This criterion means that interactions in the bulk of the plasma are more important than those at its edges, where boundary effects may take place. When this criterion is satisfied, the plasma is quasineutral.

3) Plasma frequency.

The electron plasma frequency (measuring plasma oscillations of the electrons) is large compared to the electron-neutral collision frequency (measuring frequency of collisions between electrons and neutral particles). When this condition is valid, electrostatic interactions dominate over the processes of ordinary gas kinetics.

4) Degree of ionization of plasma.

The degree of ionization of a plasma is the proportion of atoms which have lost (or gained) electrons, and is controlled mostly by the temperature. Even a partially ionized gas in which as little as 1% of the particles are ionized can have the characteristics of a plasma (i.e., response to magnetic fields and high electrical conductivity). The degree of ionization, α is defined as $\alpha = n_i/(n_i + n_a)$ where n_i is the number density of ions and n_a is the number density of neutral atoms.

(B) Why does temperature be an important factor in plasma formation? How can temperature control the degree of plasma ionization? **(15 marks)**

Answer:

temperature is an important factor in plasma formation as the kinetic energy of a plasma particle is considerably higher than its potential, where charged particles travel at high speeds. If the potential were greater than the kinetic, then the plasma state would be destroyed as the ions and electrons would want to clump together into bound states—atoms. This is why plasmas typically arise at very high temperatures.

In most cases the electrons are close enough to thermal equilibrium that their temperature is relatively well-defined, even when there is a significant deviation from a Maxwellian energy distribution function, for example, due to UV radiation, energetic particles, or strong electric fields.

Based on the relative temperatures of the electrons, ions and neutrals, plasmas are classified as "thermal" or "non-thermal". Thermal plasmas have electrons and the heavy particles at the same temperature, i.e., they are in thermal equilibrium with each other. Non-thermal plasmas on the other hand have the ions and neutrals at a much lower temperature, (normally room temperature), whereas electrons are much "hotter".

Temperature controls the degree of plasma ionization. In particular, plasma ionization is determined by the "electron temperature" relative to the ionization energy, (and more weakly by the density), in a relationship called the Saha equation. A plasma is sometimes referred to as being "hot" if it is nearly fully ionized, or "cold" if only a small fraction, (for example 1%), of the gas molecules are ionized, but other definitions of the terms "hot plasma" and "cold plasma" are common. Even in a "cold" plasma, the electron temperature is still typically several thousand degrees Celsius. Plasmas utilized in "plasma technology" ("technological plasmas") are usually cold in this sense.

(C) A volume of nitrogen gas has number density of atoms of 2.5×10^{25} m^{-3} at room temperature. Under sufficient energy, a small fraction of the gas atoms are ionized. If the number density of ions was found to be 7.5×10^{22} m^{-3}. What is the percentage of ionization in the gas?**<u>(15 marks)</u>**

<u>Answer:</u>

$n_i = 7.5 \times 10^{22}$ m^{-3}

$n_i + n_a = 2.5 \times 10^{25}$ m^{-3}

$\alpha = n_i / (n_i + n_a) = 7.5 \times 10^{22} / 2.5 \times 10^{25}$

$= 0.003 = 0.3$ %

Question (3) (55 marks)

(A) Account briefly on: **(15 marks)**

1) Plasma potential.

The potential as it exists on average in the space between charged particles, independent of the question of how it can be measured, is called the "plasma potential", or the "space potential". If an electrode is inserted into a plasma, its potential will generally lie considerably below the plasma potential due to what is termed a Debye sheath. The good electrical conductivity of plasmas causes their electric fields to be very small. This results in the important concept of "quasineutrality", which says the density of negative charges is approximately equal to the density of positive charges over large volumes of the plasma ($n_e = \langle Z \rangle n_i$), but on the scale of the Debye length there can be charge imbalance. In the special case that *double layers* are formed, the charge separation can extend some tens of Debye lengths.

2) Plasma magnetization.

Plasma in which the magnetic field is strong enough to influence the motion of the charged particles is said to be magnetized. A common quantitative criterion is that a particle on average completes at least one gyration around the magnetic field before making a collision, i.e., $\omega_{ce}/\nu_{coll} > 1$, where ω_{ce} is the "electron gyrofrequency" and ν_{coll} is the "electron collision rate". It is often the case that the electrons are magnetized while the ions are not. Magnetized plasmas are *anisotropic*, meaning that their properties in the direction parallel to the magnetic field are different from those perpendicular to it. While electric fields in plasmas are usually small due to the high conductivity, the electric field associated with a plasma moving in a magnetic field is given by $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ (where **E** is the electric field, **v** is the velocity, and **B** is the magnetic field), and is not affected by Debye shielding.

(B) Derive an expression for the potential produced by a plasma at distance r from its center? Then deduce the mathematical formula of the Debye length? **(20 marks)**

Answer:

First, consider a positive charge q all by itself. The potential at a distance r from the charge is

$$\phi = \frac{q}{4\pi\varepsilon_0 r}.$$

Now, consider a positive charge q in the middle of a plasma. It attracts electrons into its vicinity and repels positive ions. We will calculate ϕ for this case.

If we allow the particle to have both kinetic and potential energy, the probability factor becomes $\exp\left(-\frac{\frac{1}{2}mv^2 + q\phi}{kT}\right) dv_x dv_y dv_z$. ϕ depends on position so the probability depends on position.

The particle density is given by $n = \int f(v) dv_x dv_y dv_z$ so $n \propto \exp\left(-\frac{q\phi}{kT}\right)$

for electrons $n_e = n_0 \exp\left(-\frac{-e\phi}{kT}\right)$

for ions (we will suppose they are singly-ionized) $n_i = n_0 \exp\left(-\frac{e\phi}{kT}\right)$

Gauss' Law can be written as

$$\nabla . \mathbf{E} = \frac{\sigma}{\varepsilon_0}$$

$\mathbf{E} = -\nabla\phi$ so

$$-\nabla^2\phi = \frac{\sigma}{\varepsilon_0}.$$

This is *Poisson's equation.*

The charge density is $\sigma = -en_e + en_i = en_0\left(-\exp\frac{e\phi}{kT} + \exp\frac{-e\phi}{kT}\right)$.

Assume that this potential term is very small, $e\phi << kT$

$$\sigma \cong -en_0\left(1 + \frac{e\phi}{kT}\right) + en_0\left(1 - \frac{e\phi}{kT}\right) = -\frac{2n_0 e^2\phi}{kT}.$$

I am going to use spherical coordinates (and assume spherical symmetry)

$$\nabla^2\phi \equiv \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right).$$

Poisson's equation becomes

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -\frac{2n_0 e^2 \phi}{\varepsilon_0 kT}$$

with solution

$$\phi = \frac{q}{4\pi\varepsilon_0 r}\exp\left(-\frac{r}{\sqrt{\frac{\varepsilon_0 kT}{2n_0 e^2}}}\right).$$

The potential falls away exponentially.

Call $\boxed{\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}}}$ the *Debye length* then

(C) For the radio-frequency discharge in the Senior Physics Lab, T = 3 eV, n_e = 10^{17} m^{-3} and diameter about 100 mm, calculate:

1) The Debye length.
2) The plasma frequency.
3) The number of electrons in a Debye sphere. **(20 marks)**

Answer:

T = 3 eV = 3 x 11600 K

$n_e = 10^{17}$ m^{-3}

1) The Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}}$$

Take: $n_o = n_e$

Charge of an electron, e = 1.6 x 10^{-19} C

Boltzmann's constant, K = 1.38 x 10^{-23} J K^{-1}

Permittivity of free space, ε_o = 8.85 x 10^{-12} C^2 N^{-1} m^{-2}

λ_D = 4.1 x 10^{-5} m

2) The plasma frequency

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

using the above values and take ω = 2πf;

$$f_{pe} = 8.98\sqrt{n_e}$$

$= 2.8 \times 10^9$ Hz

3) The number of electrons in a Debye sphere $= 4/3\, \pi\, \lambda_D^3\, n_e$

$= 2.9 \times 10^4$

Question (4) **(50 marks)**

(A) Discuss in details the process of ionization produced by electric fields and explain with necessary graphs the principal stages of increasing the potential difference across the discharged gas? Deduce the condition of breakdown? **(20 marks)**

Answer: See the lecture notes (there are two sources for answer)

(B) Mention the different types of low-pressure discharges? **(15 marks)**

Answer:

- *Glow discharge plasmas*: non-thermal plasmas generated by the application of DC or low frequency RF (<100 kHz) electric field to the gap between two metal electrodes. Probably the most common plasma; this is the type of plasma generated within fluorescent light tubes.

- *Capacitively coupled plasma (CCP)*: similar to glow discharge plasmas, but generated with high frequency RF electric fields, typically 13.56 MHz. These differ from glow discharges in that the sheaths are much less intense. These are widely used in the microfabrication and integrated circuit manufacturing industries for plasma etching and plasma enhanced chemical vapor deposition.

- *Inductively coupled plasma (ICP)*: similar to a CCP and with similar applications but the electrode consists of a coil wrapped around the discharge volume which inductively excites the plasma.

- Wave heated plasma: similar to CCP and ICP in that it is typically RF (or microwave), but is heated by both electrostatic and electromagnetic means. Examples are helicon discharge, electron cyclotron resonance (ECR), and ion cyclotron resonance (ICR). These typically require a coaxial magnetic field for wave propagation.

(C) Calculate the root mean square velocity and the average energy (in eV) for a proton and for an electron at temperature of 10^6 K? **(15 marks)**

Answer:

$T = 10^6$ K

The root mean square velocity;

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

The average energy;

$$E_{av} = \frac{3}{2}kT$$

Using the values: Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg

Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J K^{-1}

For electron, $v_{rms} = 6.8 \times 10^6$ m/s

$E_{av} = 2.1 \times 10^{-17}$ J = 131 eV

For a proton, $v_{rms} = 1.6 \times 10^5$ m/s

$E_{av} = 2.1 \times 10^{-17}$ J = 131 eV

GOOD LUCK

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