Model Answer DSP
Final "2015"

1

(a) $x(n) = \cos\left(\frac{\pi n}{10}\right)$

$$\frac{\pi}{10} = \frac{2\pi m}{N}$$

$$\therefore \frac{m}{N} = \frac{1}{20} \quad \therefore \text{ periodic \& period} = 20$$

(b) $x(n) = \text{Im}\left[e^{\frac{jn\pi}{7}}\right] + \text{Im}\left[e^{\frac{jn}{9}}\right]$

$$\therefore \frac{m}{N} = \frac{1}{14} \rightarrow \text{periodic \& } N = 14$$

$$\frac{n}{9} = \frac{2\pi m}{N} \rightarrow \text{not periodic}$$

& overall also not periodic

2

if $x(n) = 0$ for $n < 0$ ..

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

if $x(n) = 0$ for $n < 0$

$$\therefore x_e(n) = \frac{1}{2} x(n) \quad n > 0$$

$$x_e(n) = x(n) \quad \& \ n = 0$$

So $x(n) = \begin{cases} x_e(n) & n = 0 \\ 2\,x_e(n) & n > 0 \end{cases}$

For $x_e(n) = a^n \, u(n)$

$$x(n) = \delta(n) + 2\, a^n \, u(n-1)$$

For x_{odd} we can not define $x(0)$. So it is not used

$\boxed{c}$ $x(n) = \begin{cases} 3 & n = 0 \\ 2 & n = 1 \\ 1 & n = 2 \\ 0 & \text{else} \end{cases}$

as a sum of scaled and shifted unit steps

$$x(n) = 3\,\delta(n) + 2\,\delta(n-1) + \delta(n-2)$$

however

$$\delta(n) = u(n) - u(n-1)$$

$$x(n) = 3\,[u(n) - u(n-1)] + 2\,[u(n-1) - u(n-2)] + [u(n-2) - u(n-3)]$$

$$= 3u(n) - 3u(n-1) + 2u(n-1) - 2u(n-2) + u(n-2) - u(n-3)$$

$$= 3\,u(n) - u(n-1) - u(n-2) - u(n-3)$$

$\boxed{D}$ $\boxed{a}$ $y(n) = x(n) + x(n-3) + x(n+3)$

. not causal since $x(n+3)$ Future value.

$\boxed{b}$ $y(n) = x(n) - x(n^2 - n - 1)$

non causal for n (negative)

E

a) not invirtible For $n=1$

$x(n)$ can not be Found From $y(n)$

b) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

Invertible and Corresponding to Integration

EX

$x(n) = y(n) - y(n-1)$

$y(n) - y(n-1) = \sum_{-\infty}^{n} x(k) - \sum_{-\infty}^{n-1} x(k) = x(n)$

2) Fourier analysis

A)

$H(e^{j\omega}) = 2 + 5e^{-j\omega 2} - 2e^{-3j\omega}$

directly using properties

or

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega}$$

$$= \sum_{n=-\infty}^{\infty} [2\delta(n) + 5\delta(n-2) - 2\delta(n-3)]\, e^{-jn\omega}$$

because $\sum_{n=-\infty}^{\infty} \delta(n-n_0)\, e^{-j\omega n} = e^{-jn_0\omega}$

$$\therefore H(e^{j\omega}) = 2 + 5e^{-j2\omega} - 2e^{-j3\omega}$$

$$\# \; H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-jn\omega}$$

$$= \sum_{n=2}^{\infty} a^n e^{-jn\omega}$$

put $\underline{m = n-2}$

at $n=2 \rightarrow m=0$

$n=\infty \rightarrow m=\infty$

$$\therefore \sum_{m=0}^{\infty} a^{m+2} e^{-j(m+2)\omega}$$

$$\therefore \sum_{m=0}^{\infty} a^2 a^m e^{-j2\omega} e^{-jm\omega}$$

$$= a^2 e^{-j2\omega} \sum_{0}^{\infty} \left[a e^{-j\omega}\right]^m$$

$$= a^2 e^{-j2\omega} \frac{1}{1 - a e^{-j\omega}}$$

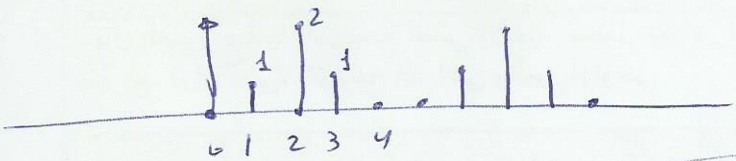
B DFS

$$\tilde{X}(k) = \sum_{n=0}^{9} \tilde{x}(n)\, e^{\frac{-j2\pi nk}{n}}$$

$$= \sum_{n=0}^{4} e^{\frac{-j2\pi nk}{n}} = \frac{1 - e^{-j\pi k}}{1 - e^{\frac{-j\pi k}{5}}}$$

$N = 5$

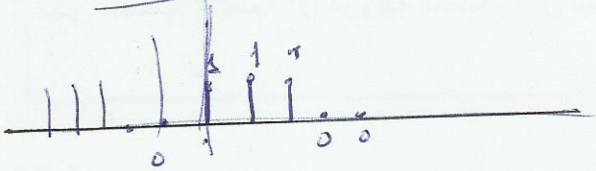
$$x(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ 2 & n=2 \\ 1 & n=3 \\ 0 & n=4 \end{cases}$$



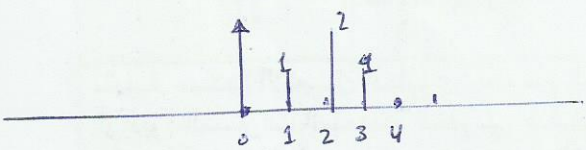
$h(n)$

$$h(n) = u(n) - u(n-3)$$

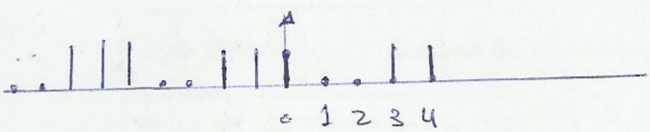
$0 \rightarrow 2$



$x(k)$



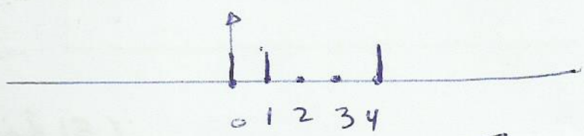
$h(-k)$



$y(0) = 0 + 0 + 0 + 1 + 0 = \boxed{1}$

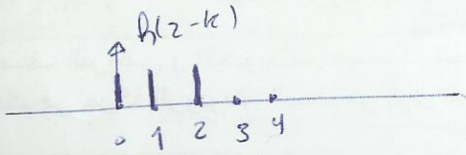
$\cancel{y(1)}$

$h(1-k)$



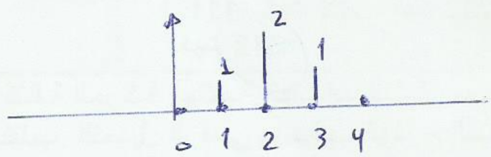
$y(1) = 0 + 1 + 0 + 0 + 0 = \boxed{1}$

$h(2-k)$

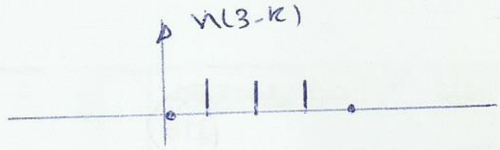


$y(2) = 0 + 1 + 2 + 0 + 0 = \boxed{3}$

$x(k)$



$h(3-k)$



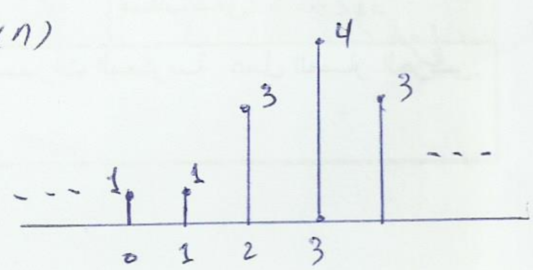
$y(3) = 0 + 1 + 2 + 1 + 0 = \boxed{4}$

$h(4-k)$

$y(4) = \cancel{0 + 2 + 1 + 0 +}$

$0 + 0 + 2 + 1 + 0 = \boxed{3}$

$y(n)$



D Find DFT

a $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$

$$= \sum_{n=0}^{N-1} a^n W_N^{nk}$$

$$= \sum_{n=0}^{N-1} (a W_N^k)^n = \frac{1-(a W_N^k)^n}{1 - a W_N^k}$$

b $x(n) = u(n) - u(n-3)$

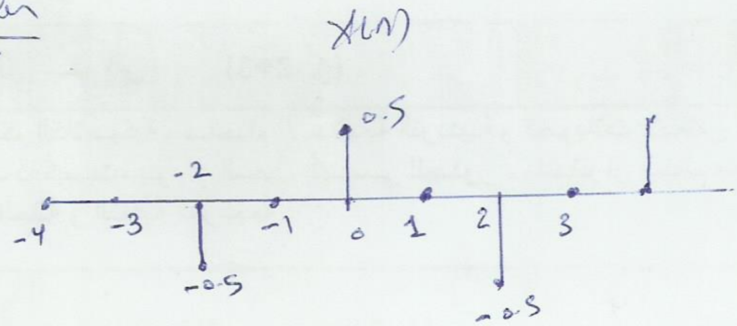
$$= \sum_{n=0}^{N-1} [u(n) - u(n-3)]\, W_N^{nk}$$

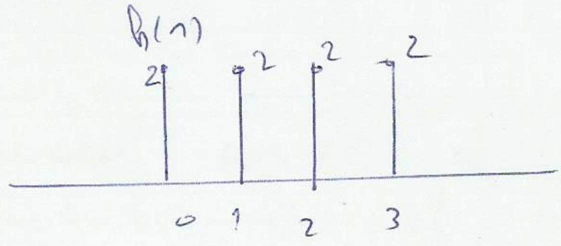
$$= \sum_{0}^{2} (W_N^k)^n = \frac{1 - W_N^{2k}}{1 - W_N^k}$$

c $X(k) = 1 + 2 W_N^{5k}$

$$= 1 + 2e^{-j\frac{2\pi 5k}{10}} = 1 + 2(-1)^k$$

circular





using table method

$$h(n) = 2\delta(n) + 2\delta(n-1) + 2\delta(n-2) + 2\delta(n-3)$$

$$\therefore y(n) = 2x(n) + 2x(n-1) + 2x(n-2) + 2x(n-3)$$

total linear

$0 \to 2$ $(0 \to 5)$

$0 \to 3$

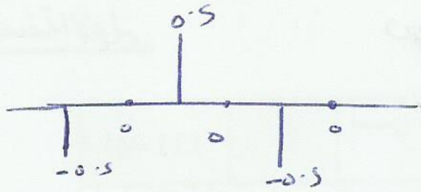
n	0	1	2	3	4	5
$2\delta(n)$	1	0	-1	0	1	0
$2x(n-1)$	0	1	0	-1	0	1
$2x(n-2)$	-1	0	1	0	-1	0
$2x(n-3)$	0	-1	0	1	0	-1
	0	0	0	0	0	0

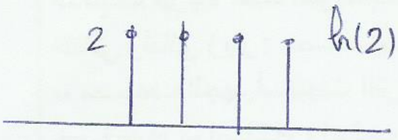
n	0	1	2	3	4	5	6	7
$y(n)$	0	0	0	0	0	0	0	0
$y(n+4)$	0	0	0	0				
	0	0	0	0				

$y(n) = 0$ For all values of n

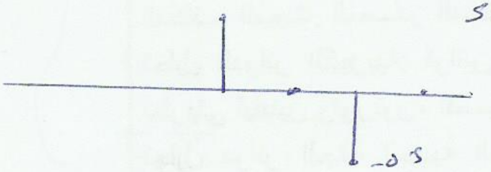
Using drawing.

X(k)





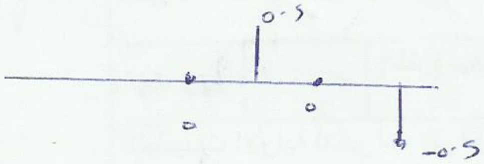
x(-k)



since x(k) even fn

$y(0) = 1 + 0 - 1 + 0 = 0$

x(1-k)



$= 0 + 1 + 0 - 1 = 0$

also all others will = 0